積分プリント解答

1 Cは積分定数とする。

(1) $\displaystyle\int\frac{dx}{x^2}=\int x^{-2}dx=\frac{1}{-2+1}x^{-2+1}+C=-x^{-1}+C=-\frac{1}{x}+C$

(2) $\displaystyle\int t^{\frac{2}{5}}dt=\frac{1}{\frac{2}{5}+1}t^{\frac{2}{5}+1}+C=\frac{5}{7}t^{\frac{7}{5}}+C$

(3) $\displaystyle\int\sqrt[4]{x^3}\,dx=\int x^{\frac{3}{4}}dx=\frac{1}{\frac{3}{4}+1}x^{\frac{3}{4}+1}+C=\frac{4}{7}x^{\frac{7}{4}}+C=\frac{4}{7}x\sqrt[4]{x^3}+C$

(4) $\displaystyle\int\frac{dx}{x^2\sqrt{x}}=\int x^{-\frac{5}{2}}dx=\frac{1}{-\frac{5}{2}+1}x^{-\frac{5}{2}+1}+C=-\frac{2}{3}x^{-\frac{3}{2}}+C=-\frac{2}{3x\sqrt{x}}+C$

2 Cは積分定数とする。

(1) $\displaystyle\int\left(4x-\frac{2}{x^3}\right)dx=\int(4x-2x^{-3})dx=4\cdot\frac{x^2}{2}-2\cdot\frac{1}{-2}x^{-2}+C=2x^2+\frac{1}{x^2}+C$

(2) $\displaystyle\int\left(4x^{\frac{1}{3}}+3x^{\frac{1}{2}}-1\right)dx=4\cdot\frac{3}{4}x^{\frac{4}{3}}+3\cdot\frac{2}{3}x^{\frac{3}{2}}-x+C=3x^{\frac{4}{3}}+2x^{\frac{3}{2}}-x+C$

(3) $\displaystyle\int\frac{x^2+x+1}{x}dx=\int\left(x+1+\frac{1}{x}\right)dx=\frac{x^2}{2}+x+\log|x|+C$

(4) $\displaystyle\int\frac{x(x+1)(x+2)}{x^3}dx=\int\frac{x^3+3x^2+2x}{x^3}dx=\int\left(1+\frac{3}{x}+\frac{2}{x^2}\right)dx=x+3\log|x|-\frac{2}{x}+C$

(5) $\displaystyle\int(\sqrt{t}+1)(\sqrt{t}-2)dt=\int(t-\sqrt{t}-2)dt=\int\left(t-t^{\frac{1}{2}}-2\right)dt$

$\displaystyle=\frac{t^2}{2}-\frac{2}{3}t^{\frac{3}{2}}-2t+C=\frac{t^2}{2}-\frac{2}{3}t\sqrt{t}-2t+C$

(6) $\displaystyle\int\left(\frac{y+2}{y}\right)^2dy=\int\left(1+\frac{2}{y}\right)^2dy=\int\left(1+\frac{4}{y}+\frac{4}{y^2}\right)dy=y+4\log|y|-\frac{4}{y}+C$

3 Cは積分定数とする。

(1) $\displaystyle\int(4\cos x+\sin x)dx=4\sin x-\cos x+C$

(2) $\displaystyle\int\left(\frac{3}{\cos^2x}-\frac{2}{\sin^2x}\right)dx=3\tan x+\frac{2}{\tan x}+C$

(3) $\displaystyle\int\left(\frac{2+\cos^3\theta}{\cos^2\theta}+\sin\theta\right)d\theta=\int\left(\frac{2}{\cos^2\theta}+\cos\theta+\sin\theta\right)d\theta$

$=2\tan\theta+\sin\theta-\cos\theta+C$

(4) $\displaystyle\int\frac{x-\cos^2x}{x\cos^2x}dx=\int\left(\frac{1}{\cos^2x}-\frac{1}{x}\right)dx=\tan x-\log|x|+C$

4 Cは積分定数とする。

(1) $\displaystyle\int(2e^x-x^2)dx=2e^x-\frac{x^3}{3}+C$

(2) $\displaystyle\int(4^x-3e^x)dx=\frac{4^x}{\log 4}-3e^x+C$

(3) $\displaystyle\int 3^{x+2}dx=9\int 3^xdx=\frac{9\cdot 3^x}{\log 3}+C=\frac{3^{x+2}}{\log 3}+C$

(4) $\displaystyle\int(e^x+2^{x+1})dx=\int(e^x+2\cdot 2^x)dx=e^x+\frac{2\cdot 2^x}{\log 2}+C=e^x+\frac{2^{x+1}}{\log 2}+C$

5 Cは積分定数とする。

(1) $\displaystyle\int\sqrt{x}(x-1)dx=\int(x\sqrt{x}-\sqrt{x})dx=\int\left(x^{\frac{3}{2}}-x^{\frac{1}{2}}\right)dx$

$\displaystyle=\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{3}x^{\frac{3}{2}}+C=\frac{2}{5}x^2\sqrt{x}-\frac{2}{3}x\sqrt{x}+C$

(2) $\displaystyle\int\sqrt{y}\left(1+\frac{1}{\sqrt{y}}\right)^2dy=\int\sqrt{y}\left(1+\frac{2}{\sqrt{y}}+\frac{1}{y}\right)dy=\int\left(\sqrt{y}+2+\frac{1}{\sqrt{y}}\right)dy$

$\displaystyle=\int\left(y^{\frac{1}{2}}+2+y^{-\frac{1}{2}}\right)dy=\frac{2}{3}y^{\frac{3}{2}}+2y+2y^{\frac{1}{2}}+C$

$\displaystyle=\frac{2}{3}y\sqrt{y}+2y+2\sqrt{y}+C$

(3) $\displaystyle\int\frac{(\sqrt{x}+1)^3}{\sqrt{x}}dx=\int\left(\frac{x\sqrt{x}+3x+3\sqrt{x}+1}{\sqrt{x}}\right)dx=\int\left(x+3x^{\frac{1}{2}}+3+x^{-\frac{1}{2}}\right)dx$

$\displaystyle=\frac{x^2}{2}+3\cdot\frac{2}{3}x^{\frac{3}{2}}+3x+2x^{\frac{1}{2}}+C=\frac{x^2}{2}+2x\sqrt{x}+3x+2\sqrt{x}+C$

(4) $\displaystyle\int\frac{(t-1)^2}{t\sqrt{t}}dt=\int\frac{t^2-2t+1}{t\sqrt{t}}dt=\int\left(t^{\frac{1}{2}}-2t^{-\frac{1}{2}}+t^{-\frac{3}{2}}\right)dt$

$\displaystyle=\frac{2}{3}t^{\frac{3}{2}}-2\cdot 2t^{\frac{1}{2}}-2t^{-\frac{1}{2}}+C=\frac{2}{3}t\sqrt{t}-4\sqrt{t}-\frac{2}{\sqrt{t}}+C$

6 Cは積分定数とする。

(1) $\displaystyle\int\left(\frac{1}{\tan x}-2\right)\sin x\,dx=\int(\cos x-2\sin x)dx=\sin x+2\cos x+C$

(2) $\displaystyle\int\frac{\cos^2x}{1+\sin x}dx=\int\frac{1-\sin^2x}{1+\sin x}dx=\int\frac{(1+\sin x)(1-\sin x)}{1+\sin x}dx$

$\displaystyle=\int(1-\sin x)dx=x+\cos x+C$

7 Cは積分定数とする。

(1) $\displaystyle\int\frac{e^{2x}-1}{e^x-1}dx=\int\frac{(e^x)^2-1}{e^x-1}dx=\int\frac{(e^x+1)(e^x-1)}{e^x-1}dx=\int(e^x+1)dx=e^x+x+C$

(2) $\displaystyle\int\frac{2^x+x\cdot 4^x}{x\cdot 2^x}dx=\int\frac{2^x+x\cdot(2^x)^2}{x\cdot 2^x}dx=\int\left(\frac{1}{x}+2^x\right)dx=\log|x|+\frac{2^x}{\log 2}+C$

8 $F'(x)=e^x-1$ より　$\displaystyle F(x)=\int(e^x-1)dx=e^x-x+C$ (Cは積分定数)

ここで，$F(1)=2$ より　$e^1-1+C=2$

よって　$C=-e+3$

したがって　$F(x)=e^x-x-e+3$

9 Cは積分定数とする。

(1) $\displaystyle\int\frac{dx}{x^7}=\int x^{-7}dx=\frac{1}{-7+1}x^{-7+1}+C$

$\displaystyle=-\frac{1}{6}x^{-6}+C=-\frac{1}{6x^6}+C$

(2) $\displaystyle\int t^{\frac{2}{5}}dt=\frac{1}{\frac{2}{5}+1}t^{\frac{2}{5}+1}+C=\frac{5}{7}t^{\frac{7}{5}}+C$

(3) $\displaystyle\int\sqrt[6]{x^5}\,dx=\int x^{\frac{5}{6}}dx=\frac{1}{\frac{5}{6}+1}x^{\frac{5}{6}+1}+C$

$\displaystyle=\frac{6}{11}x\cdot x^{\frac{5}{6}}+C=\frac{6}{11}x\sqrt[6]{x^5}+C$

(4) $\displaystyle\int\frac{1}{x\sqrt{x}}dx=\int x^{-\frac{3}{2}}dx=\frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1}+C$

$\displaystyle=-2x^{-\frac{1}{2}}+C=-\frac{2}{\sqrt{x}}+C$

10 Cは積分定数とする。

(1) $\displaystyle\int\frac{(x-2)(x-3)}{x^3}dx=\int\frac{x^2-5x+6}{x^3}dx$

$\displaystyle=\int\left(\frac{1}{x}-\frac{5}{x^2}+\frac{6}{x^3}\right)dx$

$\displaystyle=\int\left(\frac{1}{x}-5x^{-2}+6x^{-3}\right)dx$

$=\log|x|+5x^{-1}-3x^{-2}+C$

$\displaystyle=\log|x|+\frac{5}{x}-\frac{3}{x^2}+C$

(2) $\displaystyle\int\frac{(\sqrt{x}-2)^2}{x}dx=\int\frac{x-4\sqrt{x}+4}{x}dx$

$\displaystyle=\int\frac{x-4x^{\frac{1}{2}}+4}{x}dx=\int\left(1-4x^{-\frac{1}{2}}+\frac{4}{x}\right)dx$

$=x-8x^{\frac{1}{2}}+4\log|x|+C$

$=x-8\sqrt{x}+4\log|x|+C$

(3) $\displaystyle\int\frac{(t+3)^2}{\sqrt{t}}dt=\int\frac{t^2+6t+9}{t^{\frac{1}{2}}}dt=\int\left(t^{\frac{3}{2}}+6t^{\frac{1}{2}}+9t^{-\frac{1}{2}}\right)dt$

$\displaystyle=\frac{2}{5}t^{\frac{5}{2}}+6\cdot\frac{2}{3}t^{\frac{3}{2}}+9\cdot 2t^{\frac{1}{2}}+C$

$\displaystyle=\frac{2}{5}t^2\sqrt{t}+4t\sqrt{t}+18\sqrt{t}+C$

(4) $\displaystyle\int\left(4t^3-\frac{1}{t}\right)^2dt=\int\left(16t^6-8t^2+\frac{1}{t^2}\right)dt$

$\displaystyle=16\cdot\frac{t^7}{7}-8\cdot\frac{t^3}{3}-\frac{1}{t}+C$

$\displaystyle=\frac{16}{7}t^7-\frac{8}{3}t^3-\frac{1}{t}+C$

11 Cは積分定数とする。

(1) $\displaystyle\int(5\sin x-4\cos x)dx=-5\cos x-4\sin x+C$

(2) $\displaystyle\int\frac{3\cos^2x-1}{\cos^2x}dx=\int\left(3-\frac{1}{\cos^2x}\right)dx$

$=3x-\tan x+C$

(3) $\displaystyle\int\frac{5}{\sin^2x}dx=-\frac{5}{\tan x}+C$

注意 $\displaystyle\left(\frac{1}{\tan x}\right)'=-\frac{(\tan x)'}{\tan^2x}=-\frac{1}{\tan^2x}\cdot\frac{1}{\cos^2x}$

$\displaystyle=-\frac{\cos^2x}{\sin^2x}\cdot\frac{1}{\cos^2x}$

$\displaystyle=-\frac{1}{\sin^2x}$

であるから　$\displaystyle\int\frac{dx}{\sin^2x}=-\frac{1}{\tan x}+C$

(4) $\int(e^x+3^x)dx=e^x+\dfrac{3^x}{\log 3}+C$

(5) $\int(2^y\log 2-e^{y-2})dy=\int(2^y\log 2-e^{-2}\cdot e^y)dy$

$=2^y-e^{-2}\cdot e^y+C$

$=2^y-e^{y-2}+C$

12 C は積分定数とする。

(1) $\int\dfrac{\sqrt{x}+1}{x}dx=\int\dfrac{x^{\frac{1}{2}}+1}{x}dx=\int\left(x^{-\frac{1}{2}}+\dfrac{1}{x}\right)dx$

$=2x^{\frac{1}{2}}+\log|x|+C$

$=2\sqrt{x}+\log|x|+C$

参考　関数の定義域が $x>0$ であるから，$\log|x|$ を $\log x$ と表してもよい。

(2) $\int(2-\tan^2 x)dx=\int\left\{2-\left(\dfrac{1}{\cos^2 x}-1\right)\right\}dx$

$=\int\left(3-\dfrac{1}{\cos^2 x}\right)dx=3x-\tan x+C$

(3) $\int\dfrac{\cos^2 x}{\sin^2 x}dx=\int\dfrac{1-\sin^2 x}{\sin^2 x}dx$

$=\int\left(\dfrac{1}{\sin^2 x}-1\right)dx=-\dfrac{1}{\tan x}-x+C$

13 (1) $\int_4^9\dfrac{1}{\sqrt{x}}dx=\Big[2\sqrt{x}\Big]_4^9=2(3-2)=2$

(2) $\int_1^4 x^{-\frac{3}{2}}dx=\Big[-2x^{-\frac{1}{2}}\Big]_1^4=-2\left(\dfrac{1}{2}-1\right)=1$

(3) $\int_e^{e^2}\dfrac{dx}{x}=\Big[\log|x|\Big]_e^{e^2}=2-1=1$

(4) $\int_2^{e+1}\dfrac{dy}{1-y}=\Big[-\log|1-y|\Big]_2^{e+1}=-\log e=-1$

(5) $\int_0^{\pi}\sin\theta\,d\theta=\Big[-\cos\theta\Big]_0^{\pi}=-(-1-1)=2$

(6) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos t\,dt=\Big[\sin t\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=1-(-1)=2$

(7) $\int_0^{\frac{\pi}{3}}\dfrac{dx}{\cos^2 x}=\Big[\tan x\Big]_0^{\frac{\pi}{3}}=\sqrt{3}$

(8) $\int_0^1 e^{3x}dx=\left[\dfrac{1}{3}e^{3x}\right]_0^1=\dfrac{1}{3}(e^3-1)$

(9) $\int_1^2 3^x dx=\left[\dfrac{3^x}{\log 3}\right]_1^2=\dfrac{1}{\log 3}(9-3)=\dfrac{6}{\log 3}$

14 (1) $\int_1^2\dfrac{4x+1}{x^2}dx=\int_1^2\left(\dfrac{4}{x}+\dfrac{1}{x^2}\right)dx=\left[4\log|x|-\dfrac{1}{x}\right]_1^2$

$=\left(4\log 2-\dfrac{1}{2}\right)-(0-1)=4\log 2+\dfrac{1}{2}$

(2) $\int_1^e\left(\dfrac{x+1}{x}\right)^2dx=\int_1^e\left(1+\dfrac{2}{x}+\dfrac{1}{x^2}\right)dx=\left[x+2\log|x|-\dfrac{1}{x}\right]_1^e$

$=\left(e+2-\dfrac{1}{e}\right)-(1+0-1)=e+2-\dfrac{1}{e}$

(3) $\int_1^4\dfrac{x+1}{\sqrt{x}}dx=\int_1^4\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx=\left[\dfrac{2}{3}x\sqrt{x}+2\sqrt{x}\right]_1^4=\left(\dfrac{16}{3}+4\right)-\left(\dfrac{2}{3}+2\right)=\dfrac{20}{3}$

(4) $\int_1^2\dfrac{x-1}{\sqrt[3]{x}}dx=\int_1^2\left(x^{\frac{2}{3}}-x^{-\frac{1}{3}}\right)dx=\left[\dfrac{3}{5}x\sqrt[3]{x^2}-\dfrac{3}{2}\sqrt[3]{x^2}\right]_1^2$

$=\left(\dfrac{6}{5}\sqrt[3]{4}-\dfrac{3}{2}\sqrt[3]{4}\right)-\left(\dfrac{3}{5}-\dfrac{3}{2}\right)=\dfrac{3}{10}(3-\sqrt[3]{4})$

(5) $\int_0^1\left(e^{\frac{t}{2}}+e^{-\frac{t}{2}}\right)dt=\left[2e^{\frac{t}{2}}-2e^{-\frac{t}{2}}\right]_0^1=2\left(\sqrt{e}-\dfrac{1}{\sqrt{e}}\right)$

(6) $\int_1^2\dfrac{dx}{x(x-4)}=\dfrac{1}{4}\int_1^2\left(\dfrac{1}{x-4}-\dfrac{1}{x}\right)dx=\dfrac{1}{4}\Big[\log|x-4|-\log|x|\Big]_1^2$

$=\dfrac{1}{4}\left[\log\left|\dfrac{x-4}{x}\right|\right]_1^2=\dfrac{1}{4}(0-\log 3)=-\dfrac{1}{4}\log 3$

15 (1) $\int_4^9\sqrt{x}\,dx=\int_4^9 x^{\frac{1}{2}}dx=\left[\dfrac{2}{3}x^{\frac{3}{2}}\right]_4^9$

$=\dfrac{2}{3}\Big[x\sqrt{x}\Big]_4^9=\dfrac{2}{3}(27-8)=\dfrac{38}{3}$

(2) $\int_2^3\dfrac{dx}{x^3}=\int_2^3 x^{-3}dx=\left[-\dfrac{x^{-2}}{2}\right]_2^3$

$=-\dfrac{1}{2}\left[\dfrac{1}{x^2}\right]_2^3=-\dfrac{1}{2}\left(\dfrac{1}{9}-\dfrac{1}{4}\right)=\dfrac{5}{72}$

(3) $\int_0^1\sqrt[3]{t^2}\,dt=\int_0^1 t^{\frac{2}{3}}dt=\left[\dfrac{3}{5}t^{\frac{5}{3}}\right]_0^1=\dfrac{3}{5}$

(4) $\int_1^{e^2}\dfrac{dx}{x}=\Big[\log x\Big]_1^{e^2}=\log e^2=2$

(5) $\int_3^7\dfrac{dy}{y-2}=\Big[\log(y-2)\Big]_3^7=\log 5$

(6) $\int_0^{\pi}\sin\theta\,d\theta=\Big[-\cos\theta\Big]_0^{\pi}=2$

(7) $\int_0^{\frac{\pi}{6}}\dfrac{d\theta}{\cos^2\theta}=\Big[\tan\theta\Big]_0^{\frac{\pi}{6}}=\dfrac{1}{\sqrt{3}}$

(8) $\int_{-2}^0 e^{-x}dx=\Big[-e^{-x}\Big]_{-2}^0=e^2-1$

(9) $\int_0^2 3^{x-2}dx=\left[\dfrac{3^{x-2}}{\log 3}\right]_0^2=\dfrac{1}{\log 3}\left(1-\dfrac{1}{9}\right)=\dfrac{8}{9\log 3}$

16 (1) $\int_2^4\dfrac{x+1}{x^2}dx=\int_2^4\left(\dfrac{1}{x}+\dfrac{1}{x^2}\right)dx=\left[\log x-\dfrac{1}{x}\right]_2^4$

$=\left(\log 4-\dfrac{1}{4}\right)-\left(\log 2-\dfrac{1}{2}\right)$

$=2\log 2-\dfrac{1}{4}-\log 2+\dfrac{1}{2}$

$=\log 2+\dfrac{1}{4}$

(2) $\int_1^2\dfrac{5x^2-3x}{\sqrt{x}}dx=\int_1^2\left(5x^{\frac{3}{2}}-3x^{\frac{1}{2}}\right)dx=\left[5\cdot\dfrac{2}{5}x^{\frac{5}{2}}-3\cdot\dfrac{2}{3}x^{\frac{3}{2}}\right]_1^2$

$=\Big[2x^2\sqrt{x}-2x\sqrt{x}\Big]_1^2$

$=2(4\sqrt{2}-2\sqrt{2})=4\sqrt{2}$

(3) $\int_1^2\dfrac{x-1}{\sqrt[3]{x}}dx=\int_1^2\left(x^{\frac{2}{3}}-x^{-\frac{1}{3}}\right)dx=\left[\dfrac{3}{5}x^{\frac{5}{3}}-\dfrac{3}{2}x^{\frac{2}{3}}\right]_1^2$

$=\left[\dfrac{3}{5}x\sqrt[3]{x^2}-\dfrac{3}{2}\sqrt[3]{x^2}\right]_1^2$

$=\left(\dfrac{6}{5}\sqrt[3]{4}-\dfrac{3}{2}\sqrt[3]{4}\right)-\left(\dfrac{3}{5}-\dfrac{3}{2}\right)=\dfrac{3}{10}(3-\sqrt[3]{4})$

17 (1) $\int_{-1}^0(x+2)^5dx=\left[\dfrac{(x+2)^6}{6}\right]_{-1}^0=\dfrac{1}{6}(2^6-1^6)=\dfrac{21}{2}$

(2) $\int_2^6\sqrt{2x-3}\,dx=\int_2^6(2x-3)^{\frac{1}{2}}dx=\left[\dfrac{1}{2}\cdot\dfrac{2}{3}(2x-3)^{\frac{3}{2}}\right]_2^6$

$=\dfrac{1}{3}\Big[(2x-3)\sqrt{2x-3}\Big]_2^6=\dfrac{1}{3}(27-1)=\dfrac{26}{3}$

(3) $\int_0^{\frac{1}{3}}\dfrac{dx}{(3x+1)^2}=\int_0^{\frac{1}{3}}(3x+1)^{-2}dx=\left[\dfrac{1}{3}\cdot\{-(3x+1)^{-1}\}\right]_0^{\frac{1}{3}}$

$=-\dfrac{1}{3}\left[\dfrac{1}{3x+1}\right]_0^{\frac{1}{3}}=-\dfrac{1}{3}\left(\dfrac{1}{2}-1\right)=\dfrac{1}{6}$

(4) $\int_3^5\dfrac{dx}{\sqrt{2x-1}}=\int_3^5(2x-1)^{-\frac{1}{2}}dx=\left[\dfrac{1}{2}\cdot\left\{2(2x-1)^{\frac{1}{2}}\right\}\right]_3^5$

$=\Big[\sqrt{2x-1}\Big]_3^5=3-\sqrt{5}$

(5) $\int_1^3\dfrac{dx}{7-2x}=\left[-\dfrac{1}{2}\log(7-2x)\right]_1^3=-\dfrac{1}{2}(\log 1-\log 5)=\dfrac{\log 5}{2}$